Symmetry breaking and Hopf bifurcations for the planar Navier-Stokes equation

We consider the Navier-Stokes equation for an incompressible viscous fluid on a square, satisfying Navier boundary conditions and being subjected to a time-independent force. The uniqueness of stationary solutions is studied in dependence of the kinematic viscosity. For some particular forcing, it is shown that uniqueness persists on some continuous branch of stationary solutions, when the viscosity becomes arbitrarily small. On the other hand, for a different forcing, a branch of symmetric solutions is shown to bifurcate, giving rise to a secondary branch of nonsymmetric stationary solutions. Furthermore, as the kinematic viscosity is varied, the branch of symmetric stationary solutions is shown to undergo a Hopf bifurcation, where a periodic cycle branches from the stationary solution. Our proof is constructive and uses computer-assisted estimates.